Constrained and Unconstrained Flow about a Pitching Foil

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Theme

THIS paper presents a numerical method of evaluating the unsteady flows around lifting profiles in motion in a two-dimensional perfect fluid. A particular study is made of quasisteady flow about a rigid foil of RAE 104 profile in pitching oscillation about the quarter-chord axis with a zero incidence mean. The effects of flow constraint from equidistant parallel walls is presented, together with corresponding results.

Contents

In an early analysis of two-dimensional unsteady motion,[1] a potential flow theory for small-amplitude pitching oscillations of thin aerofoils made the assumption that the wake vortices moved downstream in a single plane at the velocity of the undisturbed fluid. A later analytical method included the effects of aerofoil thickness and a possible oscillation of the rear stagnation point.[2] A numerical method for unsteady two-dimensional flow of a perfect fluid around moving bodies of arbitrary cross section[3,4] was developed from a steady flow method[5,6] in which the lifting profile was represented by a distribution of source and vortex elements.

In the following numerical model, a moving profile with a defined trailing edge is represented by n uniform straight source segments of strength σ_j, $j=1...n$, arranged segmentally around the profile. In general, a circulation τ_c occurs about the profile which is represented by a distribution of m vortices of strength γ_j, $j=1,...,m$, located within the profile. The flow is considered for successive instants at times $t=0$, $t=\epsilon\Delta t$, $\epsilon=1,2,3...$, during the motion, and the following conditions are imposed at each instant:

1) The normal flow velocity component at the center I of each line source segment is equal to the corresponding velocity component of the segment.

2) The flow velocity component at the trailing edge in the direction normal to the bisector of the trailing-edge angle equals the velocity component of the trailing edge in that direction.

Hence no discontinuity is permitted between the upper and lower surface tangential velocities at the trailing edge, and the Kutta condition is satisfied at each instant. In unsteady flow, a change in the profile circulation τ_c is associated with net transport of vorticity from the profile boundary layers into the wake. This is represented in the model by the detachment of a discrete vortex element of constant strength τ_{wj} from the trailing edge during each time interval between successive instants. Hence the Kutta condition essentially is relaxed during each time interval.

After detachment, the vortex elements convect downstream to represent the wake and form an integral part of the flow calculation. The total circulation τ on a closed path taken around the foil and wake is constant and equal to the circulation at the commencement of the motion. Hence,

$$\frac{d\tau}{dt}=0 \tag{1}$$

The particular case considered is of a rigid foil at incidence θ_c to a uniform freestream velocity U. Initially, $\theta_c=\theta_{c,0}$ at the start of a quasisteady harmonic pitching oscillation. The oscillation starts from rest at maximum negative incidence and is of amplitude θ_a about mean incidence θ_m:

$$\theta_c=\theta_m-\theta_a, \qquad -\infty<t<0$$

with

$$\theta_c=\theta_m-\theta_a\cos\beta, \qquad 0<t<+\infty$$

where $\beta=\omega t$. The initial circulation about the foil is

$$\tau_c(t)=\tau_{c,0}=\tau$$

During the period $0<t<\Delta t$, the foil rotates in pitch, and the circulation about the foil changes to give a value at $t=\Delta t$ of

$$\tau_{c,1}=\tau_{c,0}+\Delta_1\tau_{c,0}$$

The generation of wake vorticity during this period is represented by the appearance of a discrete vortex element of constant strength $\tau_{w,1}=-\Delta_1\tau_{c,0}$ just downstream of the trailing edge.

During each succeeding time interval, a new vortex element is shed from the trailing edge, and the shed vortices are each convected downstream by their local flow velocities calculated at the previous time instant. At the instant $t=\epsilon\Delta t$, the number $\epsilon-1$ discrete vortices form the wake after convection from their initial positions near the trailing edge. The vorticity shed from the trailing edge during the previous time interval is $\tau_{\omega\epsilon}=\tau_{c,\epsilon-1}-\tau_{c,\epsilon}$. Application of conditions 1 and 2 at this instant yields the equations

$$\sum_{j=1}^{n}A_{ij}\sigma_{j,\epsilon}+(A_i-A_{i\omega})\tau_{c,\epsilon}=B_{i\epsilon}, \quad i=1...n \tag{2}$$

$$\sum_{j=1}^{n}A_{tj}\sigma_{j,\epsilon}+(A_t-A_{t\omega})\tau_{c,\epsilon}=B_{t\epsilon} \tag{3}$$

where $B_{i\epsilon}$, $B_{t\epsilon}$ are known coefficients given by

$$B_{i\epsilon}=-U\sin\alpha_i-\frac{d\theta_c}{dt}r_i\cos(\theta_i-\alpha_i)$$
$$-\sum_{j=1}^{\epsilon-1}B_{ij}\tau_{\omega j}-A_{i\omega}\tau_{c,\epsilon-1}$$

$$B_{t\epsilon}=-U\sin\alpha_t-\frac{d\theta_c}{dt}r_t\cos(\theta_t-\alpha_t)$$
$$-\sum_{j-1}^{\epsilon-1}B_{tj}\tau_{\omega j}-A_{t\omega}\tau_{c,\epsilon-1}$$

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The coefficients A_{ij}, $A_{i\tau}$, A_{tj}, A_t are derived from the geometry of the profile and distribution of the vorticity within the profile. For a rigid foil, they are unchanged in value from the steady-flow case. The coefficients $A_{i\omega}$, $A_{t\omega}$ are derived from the position of the new vortex $\tau_{\omega\epsilon}$, and the coefficients B_{ij}, B_{tj} are calculated from the positions of the respective wake vortices, $1 \ldots \epsilon - 1$, and thus depend on the history of the foil motion. Solution of Eqs. (2) and (3) yields values of $\sigma_{j,\epsilon}$ and $\tau_{i,\epsilon}$ and hence the irrotational velocity field at the instant $t = \epsilon\Delta t$.

Acceleration as well as velocity terms are required to determine the instantaneous pressure field. The acceleration terms are represented by $\partial\phi/\partial t$ with $\tau_c = \text{const}$ at $t = \epsilon\Delta t$, $\epsilon = 1,2,3\ldots$, from the condition of no flow around the trailing edge. In particular, the pressure is calculated at a fixed point in the flow which is instantaneously coincident with centerpoint I of a typical source segment at time $t = \epsilon\Delta t$. The pressure coefficient at I is given by

$$C_{Pi} = \frac{2}{U^2}\left\{\frac{\partial\phi_i}{\partial t} - \frac{1}{2}\left(V_i^2 - U^2\right)\right\}$$

where V_I is the resultant velocity at I.

Results were obtained for the "quasisteady" flow of an infinite fluid about an RAE 104 profile of chord C in pitching oscillation about the quarter-chord axis. This condition was attained within acceptable limits after four complete oscillations of the foil from rest. Figure 1 shows a typical lift fluctuation. Lift and corresponding circulation ratios and their respective phase angles were computed for three incidence amplitudes about a zero mean and six reduced frequencies in the range $0.16 < F < 1.73$. Pressure distributions at the foil surface were computed, and typical wake vortex displays were obtained.

The image technique was used to study the effect of wall constraint on the RAE 104 profile placed equidistantly between parallel plane walls in a uniform flow. Figure 2 shows a comparison between constrained and unconstrained unsteady flow. The influence of the walls was to increase the lift over the entire frequency range, the increase being greater than the steady flow correction at the lower reduced frequencies but less at the higher reduced frequencies. The result was a crossover between the corresponding plots of the lift ratio for the constrained and unconstrained flows. This suggested that the effects of wall constraint on unsteady oscillating flows were complex and would require careful application.

Figure 2 also shows some experimental values for an RAE 104 foil section of 127-mm chord mounted horizontally between endplates and spanning a water tunnel of 300-mm width. Top and bottom walls constrained the flow to the same configuration as that of the computed values. Corrections for buoyancy, inertia, and gravitational displacement of the foil were made. Although showing scatter, the experimental values were in fair agreement with computed values over the range of reduced frequency under study.

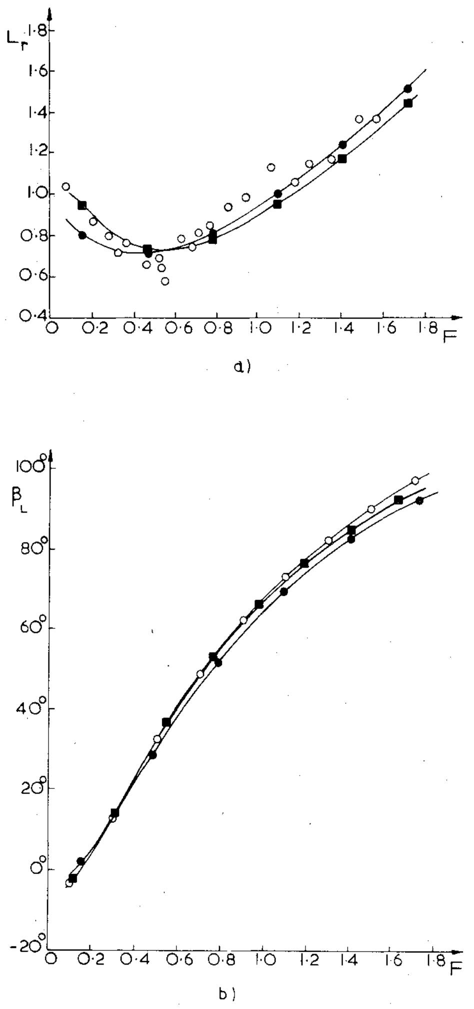

Fig. 2 Effects of flow constraint in pitching oscillation about the quarter-chord axis on a) the lift ratio L_r and b) the lift phase angle β_L, with corresponding experimental results [$\theta_m = 0$, $\theta_a = 3.2$ deg, $L_r(F_l) = L_{\max(F=F_L)}/L_{\max(F=0)}$, $F = \omega C/2U$; ■ = uniform flow constrained by two infinite plane walls parallel to the pitching axis and equidistant 1.1C from the axis (RAE 104 profile), • = uniform unconstrained flow (RAE 104 profile), ○ = experimental values for foil of RAE 104 profile, $C = 127$ mm in corresponding constrained configuration].

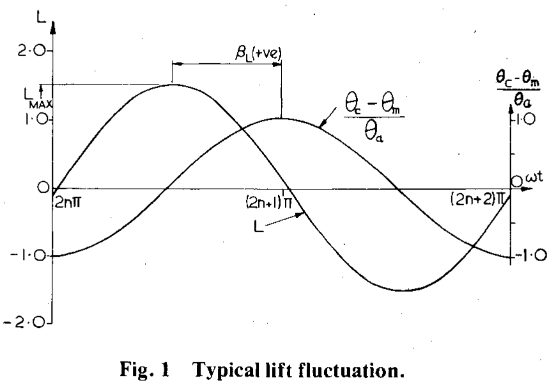

Fig. 1 Typical lift fluctuation.

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